Sharp L^2 estimates for ??-type operators and the Hardy-Littlewood inequality on H^1(??)

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Interfaces of rather different natures-as, e.g., bacterial colony or forest fire boundaries, or semiconductor layers grown by different methods (MBE, sputtering, etc.)-are self-affine fractals, and feature scaling with universal exponents (depending on the substrate's dimensionality d and global topology, as well as on the driving randomness' spatial and temporal correlations but not on the underlying mechanisms). Adding lateral growth as an essential (non-equilibrium) ingredient to the known equilibrium ones (randomness and interface relaxation), the Kardar-Parisi-Zhang (KPZ) equation succeeded in finding (via the dynamic renormalization group) the correct exponents for flat d = 1 substrates and (spatially and temporally) uncorrelated randomness. It is this interplay which gives rise to the unique, non-Gaussian scaling properties characteristic of the specific, universal type of non-equilibrium roughening. Later on, the asymptotic statistics of process h (x) fluctuations in the scaling regime was also analytically found for d = 1 substrates. For d > 1 substrates, however, one has to rely on numerical simulations. Here we review a variational approach that allows for analytical progress regardless of substrate dimensionality. After reviewing our previous numerical results in d = 1 , 2, and 3 on the time evolution of one of the functionals-which we call the non-equilibrium potential (NEP)-as well as its scaling behavior with the nonlinearity parameter ? , we discuss the stochastic thermodynamics of the roughening process and the memory of process h (x) in KPZ and in the related Golubovi?-Bruinsma (GB) model, providing numerical evidence for the significant dependence on initial conditions of the NEP's asymptotic behavior in both models. Finally, we highlight some open questions.

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